CBSE Board

Class XII Mathematics

Sample Paper - 2

Term 2 – 2021-22

Time: 2 hours **Total Marks: 40**

General Instructions:

1. *This question paper contains three sections – A, B and C. Each part is compulsory.*
2. *Section - A has 6 short answer type (SA1) questions of 2 marks each*
3. *Section – B has 4 short answer type (SA2) questions of 3 marks each.*
4. *Section - C has 4 long answer type questions (LA) of 4 marks each.*
5. *There is an internal choice in some of the questions.*
6. *Q14 is a case-based problem having 2 sub parts of 2 marks each.*

Section A

Q1 – Q6 are of 2 marks each.

1. Integrate $\int \log\left(1+x^2\right)dx$

OR

Integrate $\int \frac{\sin x}{\sin\left(x-a\right)}dx$

2. Find the sum of the order and the degree of the differential equation

$$\left(\frac{dy}{dx}\right)^2 + \frac{d}{dx}\left(\frac{dy}{dx}\right) - y = 4$$

3. If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 3 and $\frac{2}{3}$ respectively such that $\vec{a} \times \vec{b}$ is a unit vector, write the angle between $\vec{a}$ and $\vec{b}$.

4. Find the distance of the plane 3x − 4y + 12z = 3 from the origin.

5. A company has two plants to manufacturing scooters. Plant I manufactures 70% of the scooters and plant II manufactures 30%. At plant I, 30% of the scooters are rated of standard quality and at plant II, 90% of the scooters are rated of standard quality. A scooter is chosen at random and is found to be of standard quality. Find the probability that it is manufactured by plant II.

6. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "Number obtained is red". Find P(A ∩ B) if A and B are independent events.

Section B

Q7 – Q10 are of 3 marks each

7. Evaluate: $\int_0^p \frac{\sqrt{x}}{\sqrt{x}+\sqrt{p-x}}dx$

8. If $e^y(x+1)=1$, then show that $\frac{dy}{dx}=-e^y$.

OR

Obtain the differential equation of the family of circles passing through the points (a, 0) and (-a, 0).

9. Given that $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$, such that the scalar product of $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and unit vector along sum of the given two vectors $\vec{b}$ and $\vec{c}$ is unity.

10. Find the equation of the plane passing through the points (1, 2, 3) and (0, −1, 0) and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$.

OR

Find the co-ordinates of points on line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{6}$, which are at a distance of 3 units from the point (1, −2, 3).

Section C

Q11 – Q14 are of 4 marks each

11. Integrate $\int \frac{1}{x\log x\left(2+\log x\right)}dx$

12. Calculate the area between the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the x-axis between x = 0 to x = a.

OR

If AOB is a triangle in the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where OA = a and OB = b, then find the area enclosed between the chord AB and the arc AB of the ellipse.

13. Find the distance between the parallel planes $\vec{r}.\ 2i-1\hat{j}+3\hat{k}\ =4$ and $\vec{r}.\ 6\hat{i}-3\hat{j}+9\hat{k}\ +13=0$

14. Case Study

In a factory which manufactures bulbs, machines X, Y and Z manufacture 1000, 2000, 3000 bulbs, respectively. Of their outputs, 1%, 1.5% and 2 % are defective bulbs. A bulb is drawn at random and is found to be defective.

Based on the above information, answer the following question.

i. What is the probability that machine X manufactures it?

ii. What is the probability that machine Y manufactures it?

Solution

Section A

1. $I = \int \log\left(1 + x^2\right) dx$

$I = \log\left(1 + x^2\right) \int 1 dx - \int \left(\frac{d}{dx} \log\left(1 + x^2\right) \int dx\right)$

$I = x \log\left(1 + x^2\right) - \int \left(\frac{1}{1 + x^2} \times 2x \times x\right) dx + c$

$I = x \log\left(1 + x^2\right) - \int \left(\frac{2x^2}{1 + x^2}\right) dx + c$

$I = x \log\left(1 + x^2\right) - 2\int \left(\frac{x^2 + 1 - 1}{1 + x^2}\right) dx + c$

$I = x \log\left(1 + x^2\right) - 2\int \left(1 - \frac{1}{1 + x^2}\right) dx + c$

$I = x \log\left(1 + x^2\right) - 2x + 2 \tan^{-1} x + c$

OR

$I = \int \frac{\sin x}{\sin(x - a)} dx$

$I = \int \frac{\sin(x - a + a)}{\sin(x - a)} dx$

$I = \int \frac{\sin(x - a) \cos a + \cos(x - a) \sin a}{\sin(x - a)} dx$

$I = \int \left(\cos a + \tan(x - a) \sin a\right) dx$

$I = x \cos a + \sin a \log\left|\sec(x - a)\right| + c$

2. Given DE is $\left(\frac{dy}{dx}\right)^2 + \frac{d}{dx}\left(\frac{dy}{dx}\right) - y = 4$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 + \frac{d^2y}{dx^2} - y = 4$

Order is 2

Degree is 1

So, the sum is 3.

3. We know that $\sin\theta = \frac{\left|\vec{a}\times\vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|}$, where θ is the angle between $\vec{a}$ and $\vec{b}$

Since $\left|\vec{a}\right| = 3$ (given), $\left|\vec{b}\right| = \frac{2}{3}$ (given), $\left|\vec{a}\times\vec{b}\right| = 1$ (given)

$\Rightarrow \sin\theta = \frac{1}{3\times\frac{2}{3}}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sin\frac{\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}$

Thus, the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$.

4. The distance of the plane 3x − 4y + 12z − 3 = 0 from the origin (0, 0, 0) is

$= \left|\frac{3(0)-4(0)+12(0)-3}{\sqrt{(3)^2+(-4)^2+(12)^2}}\right|$

$= \left|\frac{0-0+0-3}{\sqrt{9+16+144}}\right|$

$= \left|\frac{-3}{\sqrt{169}}\right|$

$= \left|\frac{-3}{13}\right|$

$= \frac{3}{13}$

5. $P(I) = \frac{70}{100}$, $P(II) = \frac{30}{100}$

E: standard quality

$P(E/I) = \frac{30}{100}$, $P(E/II) = \frac{90}{100}$

$P(II/E) = \frac{P(II)\cdot P(E/II)}{P(I)\cdot P(E/I) + P(II)\cdot P(E/II)}$

$$= \frac{\frac{30}{100} \times \frac{90}{100}}{\frac{70}{100} \times \frac{30}{100} + \frac{30}{100} \times \frac{90}{100}}$$

$$= \frac{9}{16}$$

6. It is given that

$$P(A) = \frac{3}{6} = \frac{1}{2} \;\&\; P(B) = \frac{3}{6} = \frac{1}{2}$$

$P(A \cap B)$ = P(Numbers that are even as well as red)

= P(Number appearing is 2)

$$= \frac{1}{6}$$

Clearly, $P(A \cap B) \neq P(A) \times P(B)$

Hence, A and B are not independent events.

Section B

7. Let $I = \int_0^p \frac{\sqrt{x}}{\sqrt{x} + \sqrt{p - x}} dx$ ... (1)

According to property,

$$\int_0^a f(x)dx = \int_0^a f(a - x)dx$$

$$I = \int_0^p \frac{\sqrt{p - x}}{\sqrt{p - x} + \sqrt{x}} dx \quad ...(2)$$

Adding equations (1) and (2), we get

$$2I = \int_0^p \frac{\sqrt{x} + \sqrt{p - x}}{\sqrt{x} + \sqrt{p - x}} dx$$

$$= \int_0^p 1dx = [x]_0^p = p - 0 = p$$

Thus, $2I = p \Rightarrow I = \frac{p}{2}$

8. On differentiating $e^y(x + 1) = 1$ w.r.t x, we get

$$e^y + (x + 1)e^y \frac{dy}{dx} = 0$$

$$\Rightarrow e^y + \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -e^y$$

OR

$$x^2 + (y-b)^2 = a^2 + b^2 \text{ or } x^2 + y^2 - 2by = a^2 \quad \dots (1)$$

$$2x + 2y\frac{dy}{dx} - 2b\frac{dy}{dx} = 0$$

$$\Rightarrow 2b = \frac{2x + 2y\frac{dy}{dx}}{\frac{dy}{dx}} \quad \dots (2)$$

Substituting in (1), we get

$$(x^2 - y^2 - a^2)\frac{dy}{dx} - 2xy = 0$$

9. Given that

$$\vec{b} = 2\hat{i} + 4\hat{j} - 5k$$

$$\vec{c} = \lambda\hat{i} + 2\hat{j} + 3k$$

Now consider the sum of the vectors $\vec{b} + \vec{c}$:

$$\vec{b} + \vec{c} = \left(2\hat{i} + 4\hat{j} - 5k\right) + \left(\lambda\hat{i} + 2\hat{j} + 3k\right)$$

$$\Rightarrow \vec{b} + \vec{c} = (2+\lambda)\hat{i} + 6\hat{j} - 2k$$

Let $\hat{n}$ be the unit vector along the sum of vectors $\vec{b} + \vec{c}$:

$$\hat{n} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2k}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

The scalar product of $\vec{a}$ and n is 1. Thus,

$$\vec{a}\cdot\hat{n} = \left(\hat{i} + \hat{j} + \hat{k}\right)\cdot\left(\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}\right)$$

$$\Rightarrow 1 = \frac{1(2+\lambda) + 1\cdot 6 - 1\cdot 2}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

$$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = 2 + \lambda + 6 - 2$$

$$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = \lambda + 6$$

$$\Rightarrow (2+\lambda)^2 + 40 = (\lambda + 6)^2$$

$$\Rightarrow \lambda^2 + 4\lambda + 4 + 40 = \lambda^2 + 12\lambda + 36$$

$$\Rightarrow 4\lambda + 44 = 12\lambda + 36$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1$$

Thus, n is:

$$n = \frac{(2+1)\hat{i} + 6\hat{j} - 2k}{\sqrt{(2+1)^2 + 6^2 + 2^2}}$$

$$\Rightarrow n = \frac{3\hat{i} + 6\hat{j} - 2k}{\sqrt{3^2 + 6^2 + 2^2}}$$

$$\Rightarrow n = \frac{3\hat{i} + 6\hat{j} - 2k}{\sqrt{49}}$$

$$\Rightarrow n = \frac{3\hat{i} + 6\hat{j} - 2k}{7}$$

$$\Rightarrow n = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}k$$

10. Let the plane through (1, 2, 3) be $a(x-1)+b(y-2)+c(z-3)=0$ ...(1)

This plane is parallel to the line

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$$

$\therefore \quad a \times 2 + b \times 3 + c \times (-3) = 0$

$\Rightarrow \quad 2a + 3b - 3c = 0$ ...(2)

Also (1) passes through (0, −1, 0)

So, a + 3b + 3c = 0.............(3)

Solving (2) and (3), we get

$$\frac{a}{9+9} = \frac{b}{-3-6} = \frac{c}{6-3}$$

$$\Rightarrow \frac{a}{6} = \frac{b}{-3} = \frac{c}{1}$$

Hence the required plane is given by

$$6(x-1) - 3(y-2) + 1(z-3) = 0$$

$$\Rightarrow 6x - 3y + z = 3$$

OR

Given equation is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{6} = K$$

Any point on this line will be of the form (2K + 1, 3K − 2, 6K + 3)

Distance between points (2K + 1, 3K − 2, 6K + 3) and (1, -2, 3) is 3 units.

i.e., $\sqrt{(2K+1-1)^2 + (3K-2+2)^2 + (6K+3-3)^2} = 3$

$$\sqrt{4K^2 + 9k^2 + 36k^2} = 3$$

$$\Rightarrow 7k = 3 \Rightarrow k = \frac{3}{7}$$

∴ Required point is

$$\left(2\times\frac{3}{7}+1, 3\times\frac{3}{7}-2, 6\times\frac{3}{7}+3\right)=\left(\frac{13}{7},\frac{-5}{7},\frac{39}{7}\right)$$ is the required point.

Section C

11. $I=\int\frac{1}{x\log x(2+\log x)}dx$

Put log x = t ⇒ dx/x = dt

$$I=\int\frac{1}{t(2+t)}dt$$

Consider,

$$\frac{1}{t(2+t)}=\frac{A}{t}+\frac{B}{2+t}\quad \dots \text{(i)}$$

$$\Rightarrow \frac{1}{t(2+t)}=\frac{A(2+t)+Bt}{t(2+t)}$$

$$\Rightarrow 1=A(2+t)+Bt$$

2A + 2t + Bt = 1

2A + (2 + B)t = 1

Comparing on both sides we get

A = ½ and B = -2

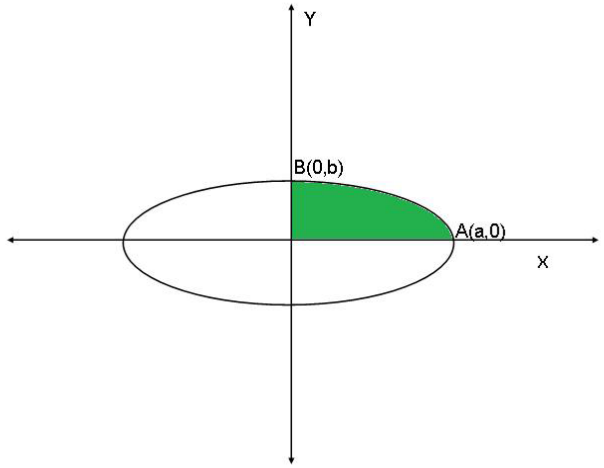
$$\Rightarrow \frac{1}{t(2+t)}=\frac{\frac{1}{2}}{t}+\frac{-2}{2+t}=\frac{1}{2t}-\frac{2}{2+t}$$

$$\Rightarrow I=\int\left(\frac{1}{2t}-\frac{2}{2+t}\right)dt$$

$$I=\frac{1}{2}\log|t|-2\log|2+t|+c$$

$$I=\frac{1}{2}\log(\log x)-2\log(2+\log x)+c$$

12.



Required area is given by

$$\int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,dx = \frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx$$

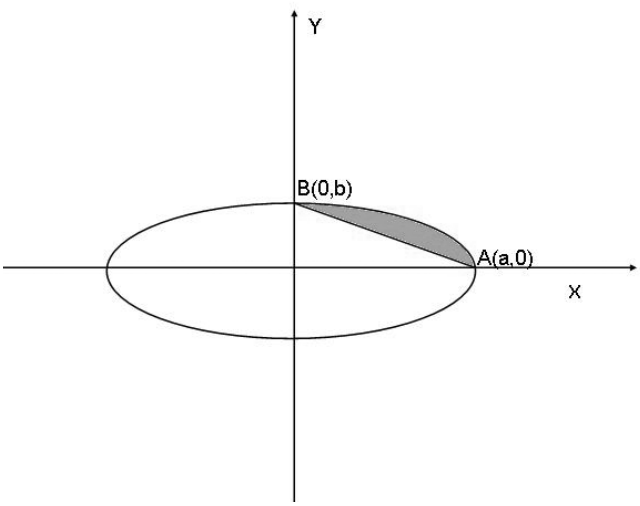
$$= \frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{b}{2a}\left[\left(0+a^2\sin^{-1}(1)\right)-\left(0+a^2\sin^{-1}(0)\right)\right]$$

$$= \frac{b}{2a}\left(a^2\times\frac{\pi}{2}\right)$$

$$= \frac{1}{4}\pi ab$$

OR



Area of triangle AOB

$$= \frac{1}{2}\times OA\times OB$$

$$= \frac{1}{2}ab$$

Now, area of ellipse in the first quadrant is given by

$$\int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,dx = \frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx$$

$$= \frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{b}{2a}\left[\left(0+a^2\sin^{-1}(1)\right)-\left(0+a^2\sin^{-1}(0)\right)\right]$$

$$= \frac{b}{2a}\left(a^2 \times \frac{\pi}{2}\right)$$

$$= \frac{1}{4}\pi ab$$

Area enclosed between the chord AB and the arc AB of the ellipse

$= \text{Area of ellipse in quadrant I} - \text{Area}(\Delta AOB)$

$$= \int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,dx - \frac{1}{2}ab$$

$$= \frac{1}{4}\pi ab - \frac{1}{2}ab$$

$$= \frac{(\pi-2)ab}{4}$$

13. Distance between the parallel planes is given by

$$\frac{|d-k|}{|\vec{n}|}$$

$$\vec{r}.\ 6\hat{i} - 3\hat{j} + 9\hat{k}\ \ + 13 = 0$$

$$\Rightarrow \vec{r}.\ 2\hat{i} - \hat{j} + 3\hat{k}\ \ = -\frac{13}{3}$$

$$\vec{r}.\ 2i - 1\hat{j} + 3\hat{k}\ \ = 4 \ \text{ and } \ \vec{r}.\ 2\hat{i} - \hat{j} + 3\hat{k}\ \ = -\frac{13}{3}$$

Therefore, the distance between the given parallel planes is

$$\frac{\left|4-\left(-\frac{13}{3}\right)\right|}{\sqrt{2\ +\ -1\ \ +3}}$$

$$= \frac{\left|4+\left(\frac{13}{3}\right)\right|}{\sqrt{4+1+9}} = \frac{\frac{25}{3}}{\sqrt{14}} = \frac{25}{3\sqrt{14}}$$

14. B_1: the bulb is manufactured by machine X

B_2: the bulb is manufactured by machine Y

B_3: the bulb is manufactured by machine Z

$P(B_1) = 1000/(1000 + 2000 + 3000) = 1/6$

$P(B_2) = 2000/(1000 + 2000 + 3000) = 1/3$

$P(B_3) = 3000/(1000 + 2000 + 3000) = 1/2$

$P(E|B_1)$ = Probability that the bulb drawn is defective, given that it is manufactured by machine X = 1% = 1/100

Similarly, $P(E|B_2)$ = 1.5% = 1.5/100 = 3/200

$P(E|B_3)$ = 2% = 2/100

i.

$$P(B_1 \mid E) = \frac{P(B_1)P(E|B_1)}{P(B_1)(PE|B_1) + P(B_2)(PE|B_2) + P(B_3)(PE|B_3)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{200} + \frac{1}{2} \times \frac{2}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{2} + 1}$$

$$= \frac{1}{1+3+6} = \frac{1}{10}$$

ii.

$$P(B_2 \mid E) = \frac{P(B_2)P(E|B_2)}{P(B_1)(PE|B_1) + P(B_2)(PE|B_2) + P(B_3)(PE|B_3)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{200}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{200} + \frac{1}{2} \times \frac{2}{100}}$$

$$= \frac{1}{\frac{1}{3} + 1 + 2}$$

$$= \frac{3}{1+3+6} = \frac{3}{10}$$